WIND MIXING CURRENTS

Office of Naval Research
Contract N7 onr 487 T.O. 3
Geophysics Branch

Navy Department
Project NR 083-061
Technical Report No. 6

John C. Freeman, Jr.
October, 1953

Research Conducted through the
Texas A&M Research Foundation
COLLEGE STATION, TEXAS
WIND MIXING CURRENTS

(Technical Report No. 6)

Project 29 is a study of the atmospheric influence on the thermal structure of the oceans, sponsored by the Office of Naval Research (Project NR 083-061, Contract N7onr-487, Task Order 3).

October, 1953

Prepared by

JOHN C. FREEMAN, JR.
ABSTRACT

A wind system may create an ocean current by differential mixing in a two layer ocean; such a current may be imposed on other currents due to the wind through effects of stress, piling up of water and mass transport by waves. In one situation studied, such differential mixing produced an average transport of water about ten to twenty percent of the transport due to wind stress.

WIND ACTION ON THE OCEAN

Sverdrup* has summarized the processes through which wind causes currents in the ocean:

a) currents directly driven by wind stress
b) currents indirectly maintained by piling up of stratified water
c) mass transport of water by wind waves.

In addition to these effects on the ocean, wind also causes mixing between the cold thermocline waters and the warm surface layer of the ocean. The following discussion shows that through such mixing action the wind may cause an additional current (in an ocean with a stable density stratification).

It will be assumed that a strong wind mixes more cold water upward into the warm mixed layer than does a weak wind. Observations tend to verify this; for example, in Figure 1 the bathy-thermograph trace of a ship at 45°N, 45°W for 8 October is contrasted

* The Oceans, 1942, pp. 489-503.
with traces for 28 September and 18 September 1949. For the period 28 September to 8 October (period A) a wind averaging Beaufort $5\frac{1}{2}$ persisted; for the period 18 September to 28 September (period B) the wind averaged Beaufort 4. During period A the mixed layer temperature cooled almost $5^\circ$ F and deepened about 110 feet, suggesting the upward mixing of a large amount of cold water into the surface layer from below the thermocline. For period B, however, the thermocline depth remained almost constant and the surface layer temperature increased, suggesting little or no mixing across the thermocline.

Consider that a strong wind (and associated mixing across the thermocline) persists in one section of the ocean, while nearby a weaker wind occurs (causing little or no mixing). The colder water in the surface layer under the stronger wind will alter the density distribution in the surface layer. In response to this new density distribution, a current will occur within the upper ocean layers.

EVALUATING THE WIND MIXING CURRENT

Assume that the ocean is a two-layer system with constant densities in the upper and lower layers. Strong winds over a portion of the ocean will then cause mixing across the thermocline (boundary) and a region of relatively cold water to be formed in the surface layer of the ocean. Neglecting the other wind effects, such a horizontal variation in the density distribution of the ocean surface layer will lead to a current.
After initial transient effects have disappeared, such a current will be in geostrophic balance so that

\[
\begin{align*}
    u &= -\frac{1}{\rho'f} \frac{\partial p}{\partial y} \\
    v &= \frac{1}{\rho'f} \frac{\partial p}{\partial x},
\end{align*}
\]

where \( u \) and \( v \) are the ocean velocity components in the \( x \)- and \( y \)-directions respectively, \( \rho' \) represents the ocean density at a given point \( (x,y) \) in the surface layer and \( p \) represents pressure.

The hydrostatic equation is

\[
p(z) = p_0 + g\rho'z,
\]

where \( p_0 \) is atmospheric pressure at the ocean surface (assumed constant here), \( g \) is gravity and \( z \) is the depth (measured positive downward). We use this equation to obtain:

\[
\begin{align*}
    u &= -\frac{gz}{\rho'f} \frac{\partial \rho'}{\partial y} \\
    v &= \frac{gz}{\rho'f} \frac{\partial \rho'}{\partial x}.
\end{align*}
\]

This whole process has not affected any pressure below the surface layer. Hence there is no current below the density discontinuity.

Thus Margules formulae for the slope of density discontinuities on a rotating earth with (4) and (5) give the relationship:

\[
\begin{align*}
    u(H) &= -\frac{\rho H \partial \rho'}{\rho' f \partial y} = -\frac{\varphi}{f} \frac{\partial \rho'}{\partial y} \\
    v(H) &= \frac{\rho H \partial \rho'}{\rho' f \partial x} = \frac{\varphi}{f} \frac{\partial \rho'}{\partial x},
\end{align*}
\]

where \( \varphi = g \frac{\partial \rho'}{\partial x} \). These expressions assert that the current in the mixed water at the interface is balanced geostrophically by the
slope of the interface. We can show that this slope is a natural result of the mixing process.

For the mixing process assumed here \( \rho' \) changes such that

\[
\Delta \rho' = \frac{\Delta H}{H} (\rho - \rho').
\]

Equation (8) tells us that since space changes exist because of mixing

\[
\frac{\partial \rho'}{\partial x} = \frac{(\rho - \rho')}{H} \frac{\partial H}{\partial x}.
\]

This is essentially the same as equation (7). Thus the slope of the interface resulting from mixing balances the geostrophic current created by mixing.

Some of the features of such a wind mixing current are shown in Figure 2. The assumptions are made that a steady wind uniform over the region is blowing over the right-hand portion, while a calm exists over the left portion of the figure. These conditions have prevailed for some time so that transient effects are no longer present. The density is constant in the vertical above and below the thermocline transition zone, although it varies in the \( x \)-direction. The induced current thus would produce the isobaric pattern shown. A geostrophic current would thus be directed into the figure, and would vary from zero at the top surface to a maximum at the deepest part of the transition zone. Figure 2 differs from Figure 1C6 in The Oceans (p. 446) in that Figure 2 shows a horizontal variation in density above the transition zone, while there is no such density transition in Figure 3 adapted from The Oceans.
ORDER OF MAGNITUDE OF THE WIND MIXING CURRENT

Since the wind mixing current varies linearly with depth in the model above the total transport of water is

\[ T_y = \frac{1}{2} \gamma H \left( \frac{\partial^2 \bar{H}}{\partial x^2} \right) \]  
\[ T_x = -\frac{1}{2} \gamma H \left( \frac{\partial^2 \bar{H}}{\partial y^2} \right) \]  

(10)

(11)

Considering data from which Figures 1 and 4 were taken* over the indicated intervals of time between Stations "C" and "D", assuming one-half the variation at "D" was due to mixing, the average transport due to wind mixing during the 10-day period 28 September to 8 October was 0.4 ft²/sec. The transport due to wind stress during this period could have been about 27 ft²/sec. Hence it would appear that the wind mixing current is small compared to possible wind stress currents. However, taking into account the direction of the wind, the resultant stress transport for this period was 2-5 ft²/sec. Thus the wind mixing current was ten to twenty percent of the net transport by the wind stress for this 10-day period.

ACKNOWLEDGMENTS

This work was stimulated by contact with Mr. H. Stommel and Mr. W. Malkus at the Conference on the Thermocline held at Big

Meadows Lodge, Virginia, 25-27 May 1953, and by work on Office of Naval Research Contract N7 onr 48703, Project NR 083-061. Messers. C. Sparger and G. Jung, members of the project, made significant contributions to the style and technical detail.
The warming during the period 18 September to 28 September occurred when the winds were Beaufort Force 4. Between 28 September and 8 October the column cooled by a large amount and the winds were Beaufort force 5 1/2.
Variations in the mixing cause horizontal variations in the density which lead to a current that increases with depth. The current at the bottom of the mixed layer must balance the slope of the thermocline if there is no current in the lower layer.
\[ p = \text{a unit of pressure} \]
\[ v = \text{a unit of velocity} \]

**Figure 2**
LEGEND

FIGURE 3

An adaptation of an illustration in Sverdrup "The Oceans" showing a current constant in the horizontal and with depth in a layer of constant density.
$P_0$

$\rho'$

NO CURRENT

$\rho'$

NO CURRENT

$P_0 + 1p$

$\rho'$

$3V$

$P_0 + 2p$

$P_0 + 3p$

$P_0 + 4p$

$P_0 + 5p$

$\rho$

$P_0 + 6p$

NO CURRENT

$P_0 + 7p$

$p$ = A UNIT OF PRESSURE

$V$ = A UNIT OF VELOCITY

FIGURE 3
LEGEND

FIGURE 4

The small amount of mixing at Station "C" is illustrated here. This is to be compared with the large amount of mixing at Station "D" for the same period.
### DISTRIBUTION LIST

<table>
<thead>
<tr>
<th>Copies</th>
<th>Addresses</th>
</tr>
</thead>
</table>
| 3      | Chief of Naval Research  
Navy Department  
Washington 25, D. C.  
Attn: Code 416 |
| 9      | Naval Research Laboratory  
Technical Services  
Washington 25, D. C. |
| 1      | Director  
Naval Research Laboratory  
Washington 25, D. C.  
Attn: Code 4010 |
| 2      | Asst. Naval Attaché for Research  
American Embassy  
Navy 100  
Fleet Post Office, New York |
| 2      | Chief, Bureau of Ships  
Navy Department  
Washington 25, D. C.  
Attn: Codes 847, 845 |
| 1      | Commander  
Naval Ordnance Laboratory  
White Oak, Silver Spring 19, Md. |
| 1      | Research & Development Board  
National Military Establishment  
Washington 25, D. C.  
Attn: Committee on Geophysics and Geography |
| 1      | Director  
Office of Naval Research  
150 Causeway Street  
Boston, Massachusetts |
| 1      | Director  
Office of Naval Research  
The John Crerar Library Building  
86 East Randolph St., 10th Floor  
Chicago 1, Illinois |
| 2      | Director  
U. S. Fish & Wildlife Service  
Department of the Interior  
Washington 25, D. C.  
Attn: Dr. L. A. Walford |
| 2      | Chief of Naval Research  
Navy Department  
Washington 25, D. C.  
Attn: Codes 466, 446 |
| 8      | U. S. Navy Hydrographic Office  
Washington 25, D. C.  
Attn: Division of Oceanography |
| 2      | Director  
U. S. Naval Electronics Laboratory  
San Diego 52, California  
Attn: Codes 550, 552 |
| 1      | California Academy of Sciences  
Golden Gate Park  
San Francisco, California  
Attn: Dr. R. C. Miller |
| 1      | Commanding General  
Research & Development Division  
Department of the Army  
Washington 25, D. C. |
| 1      | Chief, Bureau of Yards & Docks  
Navy Department  
Washington 25, D. C. |
| 1      | U. S. Fish & Wildlife Service  
450 E. Jordan Hall  
Stanford University  
Stanford, California |
| 1      | Director  
Office of Naval Research  
345 Broadway  
New York 13, N. Y. |
| 1      | Commanding Officer  
Cambridge Field Station  
230 Albany Street  
Cambridge 39, Massachusetts  
Attn: CRHSL |
| 1      | Mr. Francis M. Lucas  
ONR Resident Representative  
University of Texas  
Main Building, Room 2506  
Austin 21, Texas |
<table>
<thead>
<tr>
<th>Copies</th>
<th>Addresses</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Director</td>
</tr>
<tr>
<td></td>
<td>Office of Naval Research</td>
</tr>
<tr>
<td></td>
<td>1030 E. Green Street</td>
</tr>
<tr>
<td></td>
<td>Pasadena 1, California</td>
</tr>
<tr>
<td>1</td>
<td>Commandant (OAO)</td>
</tr>
<tr>
<td></td>
<td>U. S. Coast Guard</td>
</tr>
<tr>
<td></td>
<td>Washington 25, D. C.</td>
</tr>
<tr>
<td>1</td>
<td>Director</td>
</tr>
<tr>
<td></td>
<td>U. S. Coast and Geodetic Survey</td>
</tr>
<tr>
<td></td>
<td>Department of Commerce</td>
</tr>
<tr>
<td></td>
<td>Washington 25, D. C.</td>
</tr>
<tr>
<td>1</td>
<td>Department of Engineering</td>
</tr>
<tr>
<td></td>
<td>University of California</td>
</tr>
<tr>
<td></td>
<td>Berkeley, California</td>
</tr>
<tr>
<td>1</td>
<td>The Oceanographic Institute</td>
</tr>
<tr>
<td></td>
<td>Florida State University</td>
</tr>
<tr>
<td></td>
<td>Tallahassee, Florida</td>
</tr>
<tr>
<td>1</td>
<td>U. S. Fish &amp; Wildlife Service</td>
</tr>
<tr>
<td></td>
<td>P. O. Box 3830</td>
</tr>
<tr>
<td></td>
<td>Honolulu, T. H.</td>
</tr>
<tr>
<td>2</td>
<td>Director</td>
</tr>
<tr>
<td></td>
<td>Woods Hole Oceanographic Inst.</td>
</tr>
<tr>
<td></td>
<td>Woods Hole, Massachusetts</td>
</tr>
<tr>
<td>1</td>
<td>Director</td>
</tr>
<tr>
<td></td>
<td>Chesapeake Bay Institute</td>
</tr>
<tr>
<td></td>
<td>Box 426A, RPD 2</td>
</tr>
<tr>
<td></td>
<td>Annapolis, Maryland</td>
</tr>
<tr>
<td>1</td>
<td>Director</td>
</tr>
<tr>
<td></td>
<td>Narragansett Marine Laboratory</td>
</tr>
<tr>
<td></td>
<td>Kingston, Rhode Island</td>
</tr>
<tr>
<td>1</td>
<td>Department of Conservation</td>
</tr>
<tr>
<td></td>
<td>Cornell University</td>
</tr>
<tr>
<td></td>
<td>Ithaca, New York</td>
</tr>
<tr>
<td></td>
<td>Attn: Dr. J. Ayers</td>
</tr>
<tr>
<td>1</td>
<td>Director</td>
</tr>
<tr>
<td></td>
<td>Marine Laboratory</td>
</tr>
<tr>
<td></td>
<td>University of Miami</td>
</tr>
<tr>
<td></td>
<td>Coral Gables, Florida</td>
</tr>
<tr>
<td>1</td>
<td>U. S. Fish &amp; Wildlife Service</td>
</tr>
<tr>
<td></td>
<td>Woods Hole, Massachusetts</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Copies</th>
<th>Addresses</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Director</td>
</tr>
<tr>
<td></td>
<td>Office of Naval Research</td>
</tr>
<tr>
<td></td>
<td>1030 Geary Street</td>
</tr>
<tr>
<td></td>
<td>San Francisco 9, California</td>
</tr>
<tr>
<td>1</td>
<td>Dr. Willard J. Pierson</td>
</tr>
<tr>
<td></td>
<td>New York University</td>
</tr>
<tr>
<td></td>
<td>New York, New York</td>
</tr>
<tr>
<td>1</td>
<td>Chief of Naval Operations</td>
</tr>
<tr>
<td></td>
<td>Navy Department</td>
</tr>
<tr>
<td></td>
<td>Washington 25, D. C.</td>
</tr>
<tr>
<td></td>
<td>Attn: OP533D</td>
</tr>
<tr>
<td>1</td>
<td>U. S. Army Beach Erosion Board</td>
</tr>
<tr>
<td></td>
<td>5201 Little Falls Road, N. W.</td>
</tr>
<tr>
<td></td>
<td>Washington 16, D. C.</td>
</tr>
<tr>
<td>1</td>
<td>Allen Hancock Foundation</td>
</tr>
<tr>
<td></td>
<td>University of Southern California</td>
</tr>
<tr>
<td></td>
<td>Los Angeles 7, California</td>
</tr>
<tr>
<td>1</td>
<td>U. S. Fish &amp; Wildlife Service</td>
</tr>
<tr>
<td></td>
<td>Fort Crockett</td>
</tr>
<tr>
<td></td>
<td>Galveston, Texas</td>
</tr>
<tr>
<td>1</td>
<td>Head, Department of Oceanography</td>
</tr>
<tr>
<td></td>
<td>Brown University</td>
</tr>
<tr>
<td></td>
<td>Providence, Rhode Island</td>
</tr>
<tr>
<td>1</td>
<td>Director</td>
</tr>
<tr>
<td></td>
<td>Hawaii Marine Laboratory</td>
</tr>
<tr>
<td></td>
<td>University of Hawaii</td>
</tr>
<tr>
<td></td>
<td>Honolulu, T. H.</td>
</tr>
<tr>
<td>2</td>
<td>Head, Department of Oceanography</td>
</tr>
<tr>
<td></td>
<td>University of Washington</td>
</tr>
<tr>
<td></td>
<td>Seattle, Washington</td>
</tr>
<tr>
<td>1</td>
<td>Director</td>
</tr>
<tr>
<td></td>
<td>Lamont Geological Observatory</td>
</tr>
<tr>
<td></td>
<td>Torrey Cliff</td>
</tr>
<tr>
<td></td>
<td>Palisades, New York</td>
</tr>
<tr>
<td>1</td>
<td>Department of Zoology</td>
</tr>
<tr>
<td></td>
<td>Rutgers University</td>
</tr>
<tr>
<td></td>
<td>New Brunswick, New Jersey</td>
</tr>
<tr>
<td></td>
<td>Attn: Dr. H. H. Haskins</td>
</tr>
<tr>
<td>2</td>
<td>Director</td>
</tr>
<tr>
<td></td>
<td>Scripps Institution of Oceanography</td>
</tr>
<tr>
<td></td>
<td>La Jolla, California</td>
</tr>
<tr>
<td>Copies</td>
<td>Addresses</td>
</tr>
<tr>
<td>--------</td>
<td>-----------</td>
</tr>
</tbody>
</table>
| 1      | Bingham Oceanographic Foundation  
          Yale University  
          New Haven, Connecticut |
| 1      | The Chief, Armed Forces Special Weapons Project  
          P. O. Box 2610  
          Washington, D. C. |
| 1      | Weather Bureau  
          U. S. Department of Commerce  
          Washington 25, D. C.  
          Attn: Scientific Services |
| 2      | The Johns Hopkins University  
          Baltimore 18, Maryland  
          Attn: Librarian (1)  
          Chesapeake Bay Institute (1) |
| 3      | British Joint Services Mission  
          Main Navy Building  
          Washington 25, D. C. |
| 1      | U. S. Fish & Wildlife Service  
          S. Atlantic Offshore Fishery Investigations  
          c/o Georgia Game & Fish Commission  
          P. O. Box 312  
          Brunswick, Georgia |
| 1      | Geophysical Laboratory of  
          Department of Geology  
          Columbia University  
          New York, New York |
| 1      | Head, Dept. of Meteor. and Ocn.  
          New York University  
          New York, New York |
| 1      | Southern Regional Education Board  
          Marine Sciences  
          830 West Peachtree Street, N. W.  
          Atlanta, Georgia |
| 1      | Director  
          Marine Laboratory of the Texas Game and Fish Commission  
          Rockport, Texas |
| 1      | U. S. Navy Underwater Sound Lab  
          Attention: Dr. Marsh  
          New London, Connecticut |
| 1      | Project ARCA  
          U. S. Naval Air Station  
          Bldg. R-48  
          Norfolk, Virginia |
| 1      | Randall Laboratory of Physics  
          University of Michigan  
          Ann Arbor, Michigan  
          Attn: Dr. J. R. Fredericks |
| 1      | Department of Oceanography  
          University of Miami  
          Miami, Florida  
          Attn: F. G. Walton Smith |
| 1      | Alabama Marine Laboratory  
          Bayou La Batre, Alabama |
| 1      | Dr. Gerhard Neumann  
          Dept. of Meteo. and Ocn.  
          New York University  
          College of Engineering  
          University Heights  
          New York 51, N. Y. |
| 1      | Virginia Fisheries Laboratory  
          College of William and Mary  
          Gloucester Point, Virginia |
| 1      | Director  
          Duke University Marine Laboratory  
          Beaufort, North Carolina |
| 1      | Librarian, Wayne A. Kalenich  
          Southwest Research Institute  
          8500 Culebra Road  
          San Antonio 6, Texas |
| 1      | Director  
          University of Florida Marine Biological Station  
          Gainesville, Florida |
<table>
<thead>
<tr>
<th>Copies</th>
<th>Addresses</th>
<th>Copies</th>
<th>Addresses</th>
</tr>
</thead>
</table>
| 1      | Institute of Marine Science  
The University of Texas  
Port Aransas, Texas | 1 | Director  
Bear's Bluff Laboratories  
Wadmalaw Island, South Carolina |
| 1      | Director  
Louisiana State University  
Marine Laboratory  
Baton Rouge, Louisiana | 1 | Institute of Engineering Research  
244 Hesse Hall  
Berkeley 4, California  
Attn: Prof. J. W. Johnson |
| 1      | Director  
Institute of Fisheries Research  
University of North Carolina  
Morehead City, North Carolina | 1 | Director  
Gulf Coast Research Laboratory  
Ocean Springs, Mississippi |